

Problem Solving

Years 3-4

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'The ability to solve problems is at the heart of mathematics.'

Mathematics Counts 1982

Problem solving has always been an important but neglected element of mathematics. Far too much of mathematics teaching concentrated on the practice and consolidation of number skills in isolation from the broader context, ie why we needed to learn the skills. The introduction of the National Curriculum for mathematics, with its emphasis on 'Using and applying' was a forward step. However, since the launch of the National Numeracy Strategy and the *Framework for Teaching Mathematics* there appears to have been a decline in the emphasis on problem solving. Indeed the HMI Evaluation of the National Numeracy Strategy (2001) reports that, 'in the main teaching activity, problem solving is still underemphasised'.

This book offers 18 problem-solving lessons. Each lesson combines teaching objectives from the Solving problems section of the *Framework for Teaching Mathematics* with objectives from the general sections. This approach will provide more time within the mathematics curriculum because objectives are combined and linked. Each lesson is aimed at providing children with a challenging learning experience with the emphasis on enjoyment. Mathematics is a wonderfully exciting and rewarding subject and it is vital as teachers that we communicate this to our children.

Problem-solving qualities

In order to make progress in problem-solving activities children need to develop a range of skills or qualities that are not specific to mathematics. Many children can find problem solving very daunting because they have not developed these necessary qualities. Therefore it is vital that schools start to develop them in Key Stage 1 and continue to develop them throughout the children's school careers. Schools should discuss how they could develop these necessary qualities, which include the following.

- The ability to discuss, work cooperatively and work individually.
- The ability to communicate using mathematics.
- The ability to define and understand problems.

- The ability to think of key questions.
- The ability to explore and experiment.
- The ability to recognise 'blind alleys'.
- The ability to develop 'transfer skills'.
- The ability to use imagination and flexibility of mind.
- The ability to be reflective.
- The ability to persevere.

Mathematical problem-solving skills

In addition to the generic qualities listed above, the National Curriculum lists these skills.

Using and applying number

Pupils should be taught to:

Problem solving

- a) Approach problems involving number, and data presented in a variety of forms, in order to identify what they need to do.
- b) Develop flexible approaches to problem solving and look for ways to overcome difficulties.
- c) Make decisions about which operations and problem-solving strategies to use.
- d) Organise and check their work.

Communicating

- e) Use the correct language, symbols and vocabulary associated with number and data.
- f) Communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols.

Reasoning

- g) Present results in an organised way.
- h) Understand a general statement and investigate whether particular cases match it.
- i) Explain their methods and reasoning when solving problems involving number and data.

The lessons

Each lesson follows the same format.

Learning objectives

These are taken directly from the Yearly teaching programmes in the *Framework for Teaching Mathematics*. The solving problems objectives are linked with at least one other objective.

Vocabulary

This lists all the appropriate words and phrases to be used in the lesson. It is vital that children should see these words as well as hear them. So they should either be written on the board or, if sets of vocabulary cards are available, use these.

Resources

The lessons have been written to use a minimum of resources. Most of the resources listed would be found in most primary classrooms. Some lessons have resource sheets or activity sheets that can be photocopied. There are also three generic sheets on pages 78 to 80 that might prove useful for a variety of activities.

Oral and mental starter

These short sessions are intended to provide the children with a lively and fun start to the lesson. The objectives are taken from the National Numeracy Strategy's sample medium-term planning.

Teaching points

A detailed lesson plan to guide you through the lesson. The emphasis is on lively activities that will demonstrate to the children that mathematics is alive!

Plenary

All the plenaries have been planned to allow the children to reflect on what has gone before. Often there are 'challenges' included in the plenary. The principle here is to ensure that the children's ability is challenged but in a non-threatening way.

Support and Extension

These sections are aimed to support the less able and challenge the more able. The nature of problem solving is such that much of the work is 'open ended' and therefore differentiation should be more manageable.

Questions to guide assessment

Teacher assessment is an important component of teaching. These questions are included to help you focus on a small number of issues. The nature of problem solving is such that it is very difficult to make summative judgements. For example, 'Presents results in an organised way'. The child in Reception can do this in one way and a Y6 child in another way but both could be as valid. It is up to teachers to use their professional judgement through teacher assessment.

Calculators

Some of the lessons involve the use of calculators and in particular the OHP calculator. This is a very powerful learning tool for young children.

Wherever the calculator is suggested it is used to support children's learning. Therefore the approach in these books is in line with the recommendations of the National Numeracy Strategy.

Plan a feast

Framework for numeracy objectives

Solving problems: Problems involving 'real life', money and measures

- Solve word problems involving numbers in 'real life', money and measures, using one or more steps, including finding totals and giving change, and working out which coins to pay. Explain how the problem was solved.
- Recognise all coins and notes. Understand and use '£.p' notation (for example, know that £3.06 is £3 and 6p).

Calculations

- Extend understanding of the operations of addition and subtraction.
- Read and begin to write related vocabulary.
- Use '+', '-' and '=' signs.
- Recognise that addition can be done in any order.

VOCABULARY

altogether, total

Resources

- Photocopiable Sheets 1 and 2 (pages 42 and 43)
- Real or play coins
- OHP calculator

Oral and mental starter

Objective: Say the number that is 10 more or less, and the number that is 100 more or less, than any two-digit and three-digit number.

- Put the children into pairs. Enter a two-digit number into the OHP calculator. Ask one child in each pair to tell his or her partner which number is 10 greater than the number displayed. Ask a pair for the answer and then ask another pair to enter '+10' into the calculator. Who got the correct answer?

- Repeat this procedure with another two-digit number, but this time get the children to work out which number is 10 less than the displayed number.
- Repeat the procedure with three-digit numbers and 100 more, then 100 less.

Problem-solving challenge

Can you order a tasty meal for £15?

With the whole class

- Tell the children that they are going out for a meal at the world-famous Mega Bites café! You are such a kind teacher that you are going to give each of them £15.00 to spend on the meal. Ask them how much that is for each pair? Explain that you do not want them to have much change, so they have to spend as close to £15.00 as they can.

Children working in pairs

- Organise the children into pairs and give each pair copies of Photocopiable Sheets 1 and 2 (pages 42 and 43).
- They have to write down on Sheet 2 the following:
 - What they would choose;
 - How much it would cost;
 - What coins/notes they would use to pay for it;
 - How much change they would get.
- Stress that they need to check their calculations.

Plenary

- Ask the children questions such as:
'How much did you spend, and what on?'
'Did anyone spend exactly £15.00?'
'Did anyone have more than £1.00 change?'
- Ask further questions to discover what strategies they used to try to spend as near to £15.00 as possible, and how they checked their answers.
- Finish by giving the children this challenge:

**'Suppose I really did give you all £15.00 each.
How much would that cost me?'**

The children can discuss in pairs how to solve this problem. Stress that you are not concerned with working out the answer, only with how to get the answer.

- Take feedback from the children and then try a few methods. If multiplication has not been mentioned, show how it can be used to solve the problem.

Support

- Limit the choices the children are allowed to make from the menu. For example, tell them that they are only allowed to choose three items – a main course, a dessert and a drink.
- How many different things could they choose between from each section, and how many altogether?

Extension

- Encourage the children to make choices from the menu so that they spend as close to £15.00 as possible. Challenge them to see who can get the closest to £15.00
- Ask the children to imagine that they could eat everything on the menu. How much would that cost – firstly, to order the food, and then in indigestion tablets or a weight-loss programme!

Questions to guide assessment

- What methods did the children use to add up the amounts?
- What methods did they use to calculate the change?
- Did they check their answers and, if so, what method did they use?

Money, money, money

Framework for numeracy objectives

Solving problems

- Solve word problems involving numbers in 'real life', money and measures, using one or more steps, including finding totals and giving change, and working out which coins to pay. Explain how the problem was solved.
- Recognise all coins and notes. Understand and use '£.p' notation (for example, know that £3.06 is £3 and 6p).

Calculations

- Use informal pencil-and-paper methods to support, record or explain hundreds, tens and units (HTU) +/- tens and units (TU), and HTU +/- HTU.
- Begin to use column addition and subtraction for HTU +/- TU where the calculation cannot easily be done mentally.
- Understand multiplication as repeated addition.

VOCABULARY

altogether, multiplication, repeated addition, total

Resources

- Large display coins
- Real or play coins
- Number fans
- Number lines
- Photocopiable Sheets 3 and 4 (pages 44 and 45)
- Whiteboards (optional)

Oral and mental starter

Objective: Recall addition and subtraction facts for each number up to at least 10.

- Give the children quickfire questions, such as:
'5 + what equals 10?'
'10 subtract 2 equals what?'
- The children can respond with number fans. In order to allow the less able some 'thinking time' for this activity, tell the children that when they have worked out the answer, they should hold the number fans close to their chest until you say, 'Fish and chips'. They then show their fans. The less able children would also benefit from access to a 1-to-10 number line.

Problem-solving challenge

How many coins make a pound?

With the whole class

- Write £1.00 on the board. Ask the children how many 50p coins you would need to pay for something that cost £1.00? Write '50p + 50p' on the board.
- Repeat this activity for 20p and 10p.
- Ask the children how many 2p coins would make £1.00? Ask a child to come out and write '2p + 2p', and so on on the board, but stop them after a short while and ask:

'Who can think of a shorter way to write this information on the board?'

Encourage the children to use multiplication as a way of recording, for example we can write '5 x 2p' instead of '2p + 2p + 2p + 2p + 2p'.

- Ask the children who can think of a way of paying for something that costs £1.00 using a mixture of coins. Encourage them to approach this in a logical way, for example all the ways that use 50p.
- Tell them that they are now going to solve similar problems for different amounts of money, this time recording them on some photocopiable sheets. Organise them into pairs or let them work individually if you prefer.

Children working individually or in pairs

- On a copy of photocopiable Sheet 3 (page 44), write in the top square an amount of money, such as £2, £5 or £10. Give each child or pair one of these sheets according to their ability. They have to draw in each box the exact number of coins shown in that box to make the total you have written.
- On Sheet 4 (page 45) you write an amount again at the top but this time there are no coins drawn in the boxes. The children have to choose a selection of coins to make that total. For example, if you have written £2 at the top, they could draw the following coins: £1, 50p, 20p, 10p, 10p, 5p, 2p, 2p, 1p.

Plenary

- Ask the children to tell you some of the amounts that they found and how they recorded them. Tell them that they are going to play 'Superquick Supermarket Shopping' or SSS, as it is known. They work in pairs with pencil and paper or whiteboards. Write £5.75 on the board and tell them that you have just bought something from a shop for £1.50 but you are not sure how much change you should have. The first pair to work this out has to put a finger on their noses! Ask them to explain how they got the correct answer.
- Repeat this for other amounts and vary the questions to cope with the range of ability in your class.

Support

- Enter suitable amounts on the photocopiable sheets for these children. They will probably need coins to help them calculate appropriate amounts and may require support in recording the answers appropriately. They will also need direct teaching in how to tackle the problems logically.

Extension

- Again, enter suitable amounts on the photocopiable sheets for these children to ensure they are sufficiently challenged. Try to ensure that they record using multiplication wherever possible.

Questions to guide assessment

- Did the children tackle the problems in a logical way?
- What methods of calculation did the children use?
- Did any child do everything mentally?
- Did children use column addition?
- Which children understood the concept of multiplication?

In search of Bigfoot

Framework for numeracy objectives

Solving problems

- Solve word problems involving numbers in 'measures', using one or more steps.

Measures

- Measure and compare using standard units (metres and centimetres).
- Begin to use decimal notation for metres and centimetres.

Handling data

- Solve a given problem by organising and interpreting numerical data in simple lists, tables and graphs.

VOCABULARY

line graph, longest, shortest, tallest

Resources

- A range of measuring equipment
- Photocopiable Sheets 5 and 6 (pages 46 and 47)

Oral and mental starter

Objective: Derive doubles of whole numbers up to 20 and the corresponding halves.

- Play 'Double and Halve it Bingo'. Ask the children to draw a 3 x 3 grid on a piece of paper. They have to write any nine numbers from 1 to 20 in each square. Call out various questions such as 'Double 6' and 'Half 18'. If the children have the answer written on their bingo card they can cross it off. You may like to throw in questions such as, 'Half 17' and ask the children why they could not have such a number on their card. The first child to complete a card wins the game.

Problem-solving challenge

Do tall people have the longest feet?

With the whole class

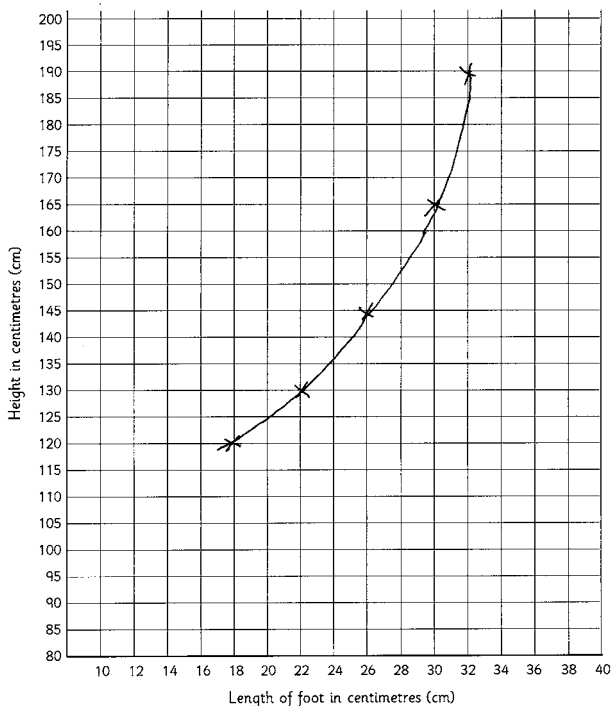
- Ask the children how many agree with this statement:
'Tall people have longer feet than shorter people.'
Take a vote amongst the class.
- Tell the children that they are going to investigate the statement. Throughout the lesson ensure that you and the children use the correct mathematical language and not words such as 'biggest' and 'largest', which could offend some children.
- Ask the children:
'Who is the tallest in the class?'
- Follow this up by asking them to arrange themselves in order of height with, say, the tallest child at the front and the shortest at the back. You may need to leave the classroom to do this!
- Then ask:
'Who has the longest feet?'
- Ask the children to see if they can get into line according to foot length, with the child with the longest feet at the front. This may lead to some confusion! Stop them and ask them to sit down. Explain that to find the answer to this question we will need to do some measuring.
- Talk to the children about how to measure the height of someone – which way you choose will depend on the measuring equipment you have – and then how to measure feet. For both these measurements the children should measure to the nearest centimetre. Show them how to record using decimal notation, for example 1.10m.

Children working in groups

- Divide the class into groups of about six. The children should take turns to measure each other's height and foot length. They can use photocopiable Sheet 5 (page 46) to record the results. Make sure that they record using appropriate decimal notation.
- When all the measurements have been completed, ask each group to write the two measurements for each child (but not the child's name) on the board.
- In order to prove the opening statement, plot the information on Sheet 6 (page 47). If you feel the children can do this themselves, give each group a copy of the sheet. The children can complete it in pairs, and this will help them check each other's work. If you feel that would be too difficult for them, then enlarge the sheet and stick it to the board or use it on an OHP or whiteboard. You can then plot the measurements for the whole class to see.

Plenary

- Discuss the results presented on the line graph and how to interpret this information. It should show a curve similar to the one below. Ask the children if it proves or disproves the statement.



- Ask the children to get back into their groups and look for the length of their feet on the activity sheet. Tell them to remember that measurement.
- Then ask them to get into a line in order of foot length, with the shortest feet first. Once the children are in order, ask them to look at each other and see whether being in this line proves or disproves the statement.

Support

- For the actual measuring activity, there are two approaches you could adopt:
 - Put the less able children together in one group and either you or a classroom assistant has them as the focus group.
 - Put the children into mixed ability groups and encourage them to work together and help each other.

Extension

- An interesting way to extend this would be for a group of children to go around the school, measuring adults; in which case, it would be worthwhile telling staff that this was going to happen!

Questions to guide assessment

- Did the children cooperate with each other and work together?
- Could the children measure with reasonable accuracy?
- Could the children record with decimal notation?
- Could they plot the information on the line graph?

The time of your life

Framework for numeracy objectives

Solving problems: Making decisions

- Choose and use appropriate operations (including multiplication and division) to solve word problems, and appropriate ways of calculating: mental, mental with jottings, pencil and paper.

Measures

- Use a calendar.

Calculations

- Use knowledge that addition can be done in any order to do mental calculations more efficiently.
- Use informal pencil and paper methods to support, record or explain $HTU \pm TU$, $HTU \pm HTU$.
- Understand multiplication as repeated addition.

VOCABULARY

day, month, week, year

Resources

- Whiteboards
- Calculators
- Photocopiable Sheet 7 (page 48)

Oral and mental starter

Objective: Order a set of three-digit numbers.

- Ask a child to choose a digit from 1 to 9 and write it on the board. Repeat this with two other children to give a three-digit number. Repeat this until you have four three-digit numbers on the board. Tell the children they have to write these numbers on their whiteboards in order, from the smallest to largest.
- Repeat this.

Problem-solving challenge

How long have you been alive?

Whole class and in pairs

- Ask the children how old they are. They will probably respond in years only.
- Explain that today they will be finding out how old they are with a little more exactness! Ask a child how old he or she is and in which month he or she was born. Ask the children to recite the months of the year.
- Model on the board how to calculate ages in years and months.
- Let the children work in pairs to calculate their ages. They can use any method they want to calculate the answer.
- Go round the class, asking the children for their ages in years and months (for example, eight years and two months). In each case, ask in which month the child was born and encourage the rest of the class to check whether the given answer is correct.
- Then ask the children to calculate their ages in whole months only, again working in pairs (ie 98 months in the example above). Revise how many months there are in a year. Most children will just use addition ($12 + 12 + 12$, and so on), but some may realise that they can use multiplication to solve the problem. Ask them for their answers and how they calculated them. If they did not use multiplication, demonstrate how to do this on the board.
- Now explain that the age in months is not exact enough, so they are going to work out how long they have been alive in weeks! Ask the children how many weeks there are in a year. They work in pairs, using photocopiable Sheet 7 (page 48) to help them.

- Ask the children for their answers and how they calculated them. Try to find as many different calculation methods as you can.
- Explain that for the final challenge they are going to calculate how many **days** they have been alive. You will be able to stifle the moans by saying that those children who want to may use calculators. Be prepared for the issue of leap years to come up. The best way to cope with this is to say that, for the purposes of this activity, leap years don't exist. Again let them use Sheet 7 to help them.

Plenary

- Tell the children that the world record for standing on one leg is 71 hours! Ask them if they can calculate how many days and hours that would be? When they have worked this out, ask how they did it. Most of them will have added or perhaps multiplied lots of 24. Demonstrate on the board how they could have done this by division – taking away multiples of 24 until the number is below 24.
- Explain that the average adult spends 62 hours a week asleep. Ask the children to calculate how many hours a night that would be. When the children have worked this out, ask how they did it.

Support

- The most obvious way to support the children in this activity is to limit the degree of accuracy of their calculation. So that while the rest of the class is working on, say, the number of days, these children would be given additional time to complete the initial tasks of calculating the number of months or weeks.

Extension

- The more able children could miss out the initial activities and proceed straight to the more advanced calculations. They could also try to solve the days problem without using a calculator! They could even be asked to work out how long they have been alive in hours, then in minutes, and finally in seconds! They will probably realise that by the time they have worked out the final calculation, it will already have been rendered inaccurate by the passage of a few more seconds!
- Another challenge would be for them to work out how many days you have been alive – provided you are happy to admit your age, that is, and deal with additional greetings cards next birthday!

Questions to guide assessment

- Did the children use efficient methods to solve the problems?
- Did any child use multiplication to solve the problem, without any prompting?
- In the plenary, did any child use division to solve the problems?