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Problem Solving Years 5-6

'The ability to solve problems is at the heart of mathematics.'

Mathematics Counts 1982

Problem solving has always been an important but neglected element of mathematics. Far too much of mathematics teaching concentrated on the practice and consolidation of number skills in isolation from the broader context, ie why we needed to learn the skills. The introduction of the National Curriculum for mathematics, with its emphasis on 'Using and applying', was a forward step. However, since the launch of the National Numeracy Strategy and the *Framework for Teaching Mathematics* there appears to have been a decline in the emphasis on problem solving. Indeed the HMI Evaluation of the National Numeracy Strategy (2001) reports that 'in the main teaching activity, problem solving is still underemphasised'.

This book offers 18 problem-solving lessons. Each lesson combines teaching objectives from the Solving Problems section of the *Framework for Teaching Mathematics* with objectives from the general sections. This approach will provide more time within the mathematics curriculum because objectives are combined and linked. Each lesson is aimed at providing children with a challenging learning experience with the emphasis on enjoyment. Mathematics is a wonderfully exciting and rewarding subject and it is vital as teachers that we communicate this to our children.

Problem-solving qualities

In order to make progress in problem-solving activities children need to develop a range of skills or qualities that are not specific to mathematics. Many children can find problem solving very daunting because they have not developed these necessary qualities. Therefore it is vital that schools start to develop them in Key Stage 1 and continue to develop them throughout the children's school careers. Schools should discuss how they could develop these necessary qualities, which include the following:

- The ability to discuss, work cooperatively and work individually.
- The ability to communicate using mathematics.
- The ability to define and understand problems.
- The ability to think of key questions.
- The ability to explore and experiment.
- The ability to recognise 'blind alleys'.
- The ability to develop 'transfer skills'.
- The ability to use imagination and flexibility of mind.
- The ability to be reflective.
- The ability to persevere.

Mathematical problem-solving skills

In addition to the generic qualities listed above, the National Curriculum lists these skills.

Using and applying number

Pupils should be taught to:

Problem solving

a) Make connections in mathematics and appreciate the need to use numerical skills and knowledge when solving problems in any other parts of the mathematics curriculum.
b) Break down a more complex problem or calculation into simpler steps before attempting a solution; identify the information needed to carry out the tasks.
c) Select and use appropriate mathematical equipment, including ICT.
d) Find different ways of approaching a problem in order to overcome any difficulties.
e) Make mental estimates of the answers to calculation; check results.

Communicating

f) Organise work and refine ways or recording.
g) Use notation diagrams and symbols correctly within a given problem.
h) Present and interpret solutions in the context of the problem.
i) Communicate mathematically, including the use of precise mathematical language.

Reasoning

j) Understand and investigate general statements.
k) Search for patterns and their results; develop logical thinking and explain their reasoning.

The lessons

Each lesson follows the same format.

Learning objectives
These are taken directly from the yearly teaching programmes in the *Framework for Teaching Mathematics*. The solving problems objectives are linked with at least one other objective.

Vocabulary
This lists all the appropriate words and phrases to be used in the lesson. It is vital that children should see these words as well as hear them. So either they should be written on the board or if sets of vocabulary cards are available use these.

Resources
The lessons have been written to use a minimum of resources. Most of the resources listed would be found in most primary classrooms. Some lessons have resource sheets or activity sheets that can be photocopied.

Oral and mental starter
These short sessions are intended to provide the children with a lively and fun start to the lesson. The objectives are taken from the National Numeracy Strategy's sample medium-term planning.

Teaching points
A detailed lesson plan to guide you through the lesson. The emphasis is on lively activities that will demonstrate to the children that mathematics is alive!

Plenary
All the plenaries have been planned to allow the children to reflect on what has gone before. Often there are 'challenges' included in the plenary. The principle here is to ensure that the children's ability is challenged but in a non-threatening way.

Support and Extension
These sections are aimed at supporting the less able and challenging the more able. The nature of problem solving is such that much of the work is 'open ended' and, therefore, differentiation should be more manageable.

Questions to guide assessment
Teacher assessment is an important component of teaching. These questions are included to help you focus on a small number of issues. The nature of problem solving is such that it is very difficult to make summative judgements. For example, 'Presents results in an organised way'. The child in Reception can do this in one way and a Y6 child in another way but both could be as valid. It is up to teachers to use their professional judgement through teacher assessment.

Calculators

Some of the lessons involve the use of calculators and in particular the OHP calculator. This is a very powerful learning tool for young children.

Wherever the calculator is suggested it is used to support children's learning. Therefore the approach in these books is in line with the recommendations of the National Numeracy Strategy.

YEAR 5

Cheap travel

Framework for Numeracy objectives

Solving problems

- Use all four operations to solve simple word problems involving numbers and quantities based on 'real life' and money using one or more steps. Explain methods and reasoning.

Calculations

- Use informal pencil and paper methods to support, record or explain additions and subtractions. Extend written methods to column addition and subtraction of two integers less than 10,000 and addition of more than two integers less than 10,000.
- Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions.
- Extend written methods to: short multiplication of HTU, long multiplication of TU by TU and short division of HTU by U.
- Develop calculator skills and use a calculator effectively.
- Check with the inverse operation when using a calculator.

VOCABULARY

total, altogether, average, difference

Resources

- Photocopiable Sheets 1 to 5 (pages 42 to 46)
- A class set of calculators

Oral and mental starter

Objective: Add and subtract any pair of two-digit numbers, including crossing 100.

- Give the children 'target numbers', for example 124. They have to write down as many TU and TU additions as they can that equal that number. Alternatively you can ask the children to write down, say five additions. Repeat this for other target numbers.
- Extend this to asking the children what they would need to subtract from the target number to leave a TU number.

Problem-solving challenge

Which is the cheapest method of travel?

With the whole class

- Give the children copies of photocopiable Sheet 1 (page 42). Read through the sheet with them, asking the children how they would fill in the answers for each section for the journey from Abouttown to Bettertown. Demonstrate on the board how to calculate the cost of each journey. Make a point of stressing the use of multiplication rather than repeated addition.
- Ask the children to tell what 'total' means and how you calculate it. Ask the children what the term 'average' means and how you calculate it.

Children working individually

- Give the children copies of photocopiable Sheet 2 (page 43). Tell them that this sheet is for recording their calculations for the challenges on Sheet 1 but that they should write their answers on Sheet 1.
- Tell them to check their calculations by an appropriate method; this would include checking by

using a calculator or the inverse operation. Whatever method they choose they should write the way they did it in the box.

- Now give the children photocopiable Sheet 3 (page 44), which is the same activity but for a family of four who are doing a return journey. Let them use calculators to calculate the answers to this sheet but encourage them to check with the calculator by using the inverse operation.

Plenary

- Ask the children what calculation methods they used to find the answers. Discuss with them how realistic they think the answers are. Explain that in real life the journey by train for four people would be a lot cheaper because train companies offer special family tickets.
- To finish the lesson give the children some 'difference' questions to calculate in pairs. For example, 'What is the difference in miles between the distances from A to B and C to D?'

Support

- Let the children use photocopiable Sheet 4 (page 45). This has easier calculations and therefore lets the children take part in the same type of problem but at their level.

Extension

- The children could be set harder challenges, such as 'What would be the cost of each return journey for two families? One family has three people in it and the other has four people.'
- The children could use the ideas on photocopiable Sheet 1 but have to design their own version of it. They have to think of the distances and costs themselves. Let them use photocopiable Sheet 5 (page 46) as a template.

Answers

Photocopiable sheet 1

Town	Town	Distance in miles	Car cost	Taxi cost	Train cost
Abouttown	Bettertown	153	13.77	30.60	7.65
Coldtown	Deartown	207	18.63	41.40	10.35
Everytown	Fasttown	504	45.36	100.80	25.20
Greattown	Hottown	801	72.09	160.20	40.05
Idealtown	Jollytown	945	85.05	189.00	47.25
		Totals	234.90	522.00	130.50
		Average Costs	**46.98**	**104.40**	**26.10**

Photocopiable Sheet 3

Town	Town	Distance in miles	Car cost	Taxi cost	Train cost
Abouttown	Bettertown	153	27.54	612.00	36.72
Coldtown	Deartown	207	37.26	828.00	49.68
Everytown	Fasttown	504	90.72	2016.00	120.96
Greattown	Hottown	801	144.18	3204.00	192.24
Idealtown	Jollytown	945	170.10	3780.00	226.80
		Totals	469.80	10440.00	626.40
		Average Costs	**93.96**	**2088.00**	

125.28

Photocopiable Sheet 4

Town	Town	Distance in miles	Car cost	Taxi cost	Train cost
Abouttown	Bettertown	10	.80	20.00	.50
Coldtown	Deartown	20	1.60	40.00	1.00
Everytown	Fasttown	30	2.40	60.00	1.50
Greattown	Hottown	40	3.20	80.00	2.00
Idealtown	Jollytown	50	4.00	100.00	2.50
		Totals	**12.00**	**300.00**	**7.50**

Questions to guide assessment

- Did the children choose appropriate calculation methods?
- Did the children check their calculations with an appropriate method?
- Did the children use the calculator effectively?

YEAR 5

Holiday on the cheap!

Framework for Numeracy objectives

Solving problems

- Use all four operations to solve simple word problems involving numbers and quantities based on 'real life' and money using one or more steps, including making simple conversions of pounds to foreign currency and finding simple percentages.
- Explain methods and reasoning.

Calculations

- Use informal pencil and paper methods to support, record or explain additions and subtractions. Extend written methods to column addition/subtraction of two integers less than 10,000 and addition of more than two integers less than 10,000. Addition or subtraction of a pair of decimal fractions.
- Approximate first. Use informal pencil and paper methods to support, record or explain multiplications and divisions.
- Extend written methods to: short multiplication of HTU, long multiplication of TU by TU and TU by T and short division of HTU by U.
- Develop calculator skills and use a calculator effectively.
- Check with the inverse operation when using a calculator.

VOCABULARY

deduct, convert, conversion, exchange rate, supplements

Resources

- Number fans
- A range of holiday brochures. (You could ask the children to get some. Ideally they should go to the travel agent's with an adult.)
- Photocopiable Sheet 6 (page 47)

Oral and mental starter

Objective: Recall addition and subtraction facts for each number up to 20.

- Give the children number fans. Set them questions such as, 'If I subtract 6 from 20, what would I add to 6 to get back to 20?' and 'I subtracted a number from 20 and I have 12 left; what was that number?'

Problem-solving challenge

Which is the cheapest holiday?

With the whole class

- Organise the children into pairs. Give each pair a travel agent's holiday brochure.
- Give the children time to look through the brochures before directing them to the sections that show the prices. Using one brochure as an example go through the 'pricing policy', ie they try to make it as complicated as they can! Explain that there are things called supplements, which have to be added to the price given in the brochure. These can be for: the time of year you go on holiday; where you fly from; how many people are in the group; whether there are children going and so on.
- Explain to the children that they are going to work out the costs of some holidays today but, luckily for them, you have invented your own hotels and prices!
- Give the children copies of photocopiable Sheet 6 (page 47). Explain that the prices given are in euros. Some explanation of the euro may be necessary and it would be ideal to show them some. Demonstrate on the board how to convert euros to pounds. If the children experience difficulty with this idea you could construct a conversion chart on the board.

  €1: £0.60 €2: £1.20 €3: £1.80

- Rather than build up the whole chart just put up important facts like €50, €100 and so on.
- Depending on the objectives you want to set for this lesson the children could do all the calculations with the calculator. Or you may wish to 'mix and match' the objectives and allow the children to use the calculator for some of the harder calculations.
- Give the children these challenges.

Children working in pairs

Challenge 1

How much would it cost in pounds for one person to go for one week's holiday (Saturday to Saturday, not including the last Saturday night) in each hotel below? This person wants breakfast every day.
Answers:

Hotel Sunny Delight – 7 nights @ €50 = €350, deduct €20 = €330 or £198

Hotel Happy – 7 nights @ €40 = €280, add 7 breakfasts @ €7 = €49 = €329, deduct €16 = €313 or £187.80

Hotel Goodtime – 7 nights @ €55 = €385, deduct €60 = €325 or £195

Challenge 2

How much would it cost in pounds for two people to go for a week's holiday (Saturday to Saturday, not including the last Saturday night) in each hotel? The two people want to share a double room and have breakfast every day.
Answers:

Hotel Sunny Delight
7 nights @ €70 = €490, deduct €20 = €470 or £282.00

Hotel Happy
7 nights @ €60 = €420, add breakfasts €98 (14 @ €7) = €518, deduct €16 = €502 or £301.20

Hotel Goodtime
7 nights @ €75 = €525, deduct €60 = €465 or £279

Challenge 3

The final challenge for the children is to announce that each hotel has agreed a 10% reduction in their prices! Ask them to work out the new prices in euros.

Plenary

- Discuss with the children why the holiday brochures are so confusing. Ask the children to discuss, in pairs, a way of pricing holidays to make it simpler to understand.
- Take feedback from the children and discuss the merits of some of their suggestions.

Support

- Limit the number of days that the children have to work out the price for and just ask the children to write the answer in euros and not convert the price into pounds.

Extension

- There are many ways to extend this activity. The number of days of each holiday can be increased as can the number of people going on holiday. Increase the percentage reduction to a figure the children will find challenging. Ask the children to work out what the VAT might be on each holiday.
- Alternatively, see if the children can use the brochures to work out the cost of a holiday for four somewhere in August, for example.

Questions to guide assessment

- Did the children choose appropriate calculation methods?
- Did the children check their calculations with an appropriate method?
- Did the children use the calculator effectively?

YEAR 5

When is a rectangle not a rectangle?

Framework for Numeracy objectives

Solving problems

- Make and investigate a general statement about shapes by finding examples that satisfy it.
- Explain methods and reasoning, orally and in writing.

Shape and space

- Recognise properties of rectangles.
- Understand and use angle measure in degrees.

VOCABULARY

diagonals, bisect, parallel, symmetry

Resources

- Protractors or angle measures
- Plain paper
- Scissors
- Rulers
- Mirrors
- A range of rectangles
- Photocopiable Sheet 7 (page 48)

Oral and mental starter

Objective: Halve any two-digit number.

- Play 'Halving bingo'. Ask the children to draw a 4 x 4 table and write in it 16 numbers below 50. Tell them that they can have halves if they want to. For example $12\frac{1}{2}$. You then start asking calculation questions such as, 'Half of 70?' and 'Half of 89?' The children have to work out the answer and if they have that number on their 'bingo card' they cross it out. The first child to complete their card wins.

Problem-solving challenge

With the whole class

- Organise the children into pairs and give each pair a copy of photocopiable Sheet 7 (page 48). Before sending them off to work in pairs, explain to them what they are going to do. They are going to read each statement on the sheet and then vote on the statement, either agreeing or disagreeing. Remind them what 'bisect' means and make sure they understand 'four lines of symmetry'. Say they are going to investigate each statement. The onus is on them to prove or disprove each statement. The way they can do this is by drawing rectangles and investigating each statement in turn. The 'proof' they offer can be mainly drawn but must have some written explanation.
- Before they start, remind the children how to draw a rectangle using rulers and protractors.

Children working in pairs

- As the children are investigating each statement try not to intervene unless they are going in a hopeless direction with their investigation. They will, no doubt, ask if their 'proofs' are acceptable but try not to be drawn.
- Either stop the children after all of them have finished investigating the first statement and have a mini-plenary where the statement is discussed, or wait until everyone has investigated all the statements and discuss them in the plenary. Tell the pairs that they then have to choose one of the statements and write on the bottom part of Sheet 7 whether or not they agree with it.

Plenary

- Use these questions to guide the plenary or the mini-plenaries you had in the main part of the lesson.
  - Do you agree with the statement?
  - Does anyone disagree with the statement and why?
  - What kind of proof have you offered for each statement?
- Make this statement to the children: 'All squares are rectangles'. Ask them to work in pairs to investigate this statement by drawing a square and discussing it by comparing the square to each of the statements on Sheet 7. Ask for feedback from the children.
- If necessary, finish the lesson by telling the children that all the statements are true.

Support

- Let the children draw around rectangles rather than draw them with a protractor.
- Limit the statements they have to investigate to those involving angles and symmetry.

Extension

- Ask the children to investigate the properties of other shapes, such as triangles and pentagons. Ask them to make a list of properties similar to the statements for a rectangle.

Questions to guide assessment

- Could the children investigate each statement in a logical way?
- Could they explain their methods and reasoning?
- Could they draw the rectangles accurately?

YEAR 5

Name that triangle!

Framework for Numeracy objectives

Solving problems

- Make and investigate a general statement about shapes by finding examples that satisfy it.
- Explain methods and reasoning, orally and in writing.

Shape and space

- Classify triangles (isosceles, equilateral, scalene), using criteria such as equal sides, equal angles, and lines of symmetry.

VOCABULARY

isosceles, equilateral, scalene, lines of symmetry

Resources

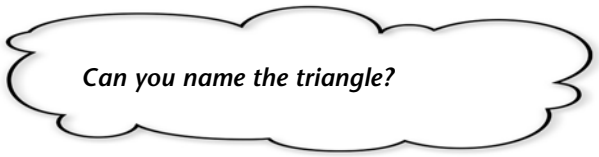
- Photocopiable Sheets 8 to 10 (pages 49 to 51)
- Scissors
- Rulers
- Protractors
- A range of 2D triangles

Oral and mental starter

Objective: Read and write whole numbers to at least 100,000.

- Ask five different children to tell you a digit from 1 to 9. Write the digits on the board to make a five-digit number. The children have to work in pairs and tell each other what number it is. Repeat this process a few times.
- Then tell the children that you are going to say a number and they have to write it down and then show each other their answer. They have to say to each other whether the answer is right or wrong. Repeat this a few times.

Problem-solving challenge



With the whole class

- Ask the children to tell you the names of some triangles. At this age they will probably know the names of an equilateral triangle and an isosceles triangle. Ask them what the properties of each triangle are. (An equilateral triangle has equal angles and sides. An isosceles triangle has two equal angles and sides.) Draw these two on the board.
- Now draw on the board a scalene triangle and tell the children its name. Ask what they can tell you about it. (It has no equal sides or angles.)
- Do the same for a right-angled triangle.

Children working in pairs/individually

- Give the children copies of photocopiable Sheet 8 (page 49). Tell them that they have to cut out each statement and triangle name card and then match the statements to a type of triangle. Discuss anything that needs clarification, such as what lines of symmetry are.
- When they have done this they can be given photocopiable Sheet 9 (page 50). They now have to cut out each shape and then, using rulers and protractors, measure the angles and sides of each triangle and write the measurements on the actual triangle.
- When the children have completed this task, explain to them that the next challenge is to record the identity of different triangles together with a general statement about each one. To do this give them copies of photocopiable Sheet 10 (page 51). They should sketch the triangle and write one statement about it.

Plenary

- Ask the children to read out their statements about their triangles. Talk about the lines of symmetry in a triangle. For some triangles it is quite straight forward to identify lines of symmetry. How could we find a line of symmetry in a scalene triangle? Indeed, is this possible?
- Show the children an equilateral triangle and ask them what size the angles are. Then ask them to imagine that you have made the triangle twice as large. Ask them what size the angles would be now. Repeat this for other triangles to ensure that the children understand that no matter what type of triangle, if you make it uniformly larger or smaller the angles remain the same.

Support

- Give the children a range of triangles to sort using some, but not all, of the statements on photocopiable Sheet 8. They could be given, for example, 'All the sides are of equal length,' and they have to find as many triangles as they can that fit that criterion. Change the criteria you give the children depending on their ability.

Extension

- Ask the children to write as many different criteria as they can for all the triangles.

Questions to guide assessment

- Could the children classify their triangles correctly?
- Could they write precise statements about the properties of triangles?
- Could the children identify lines of symmetry?