CONTENTS

Problem Solving KS1

'The ability to solve problems is at the heart of mathematics.'

Mathematics Counts 1982

Problem solving has always been an important but neglected element of mathematics. Far too much of mathematics teaching concentrated on the practice and consolidation of number skills in isolation from the broader context, ie why we needed to learn the skills. The introduction of the National Curriculum for mathematics, with its emphasis on 'Using and applying' was a forward step. However since the launch of the National Numeracy Strategy and the *Framework for Teaching Mathematics* there appears to have been a decline in the emphasis on problem solving. Indeed the HMI Evaluation of the National Numeracy Strategy (2001) reports that, 'in the main teaching activity, problem solving is still underemphasised'.

This book offers 18 problem-solving lessons. Each lesson combines teaching objectives from the Solving problems section of the *Framework for Teaching Mathematics* with objectives from the general sections. This approach will provide more time within the mathematics curriculum because objectives are combined and linked. Each lesson is aimed at providing children with a challenging learning experience with the emphasis on enjoyment. Mathematics is a wonderfully exciting and rewarding subject and it is vital as teachers that we communicate this to our children.

Problem solving qualities

In order to make progress in problem-solving activities children need to develop a range of skills or qualities that are not specific to mathematics. Many children can find problem solving very daunting because they have not developed these necessary qualities. Therefore it is vital that schools start to develop them in Key Stage 1 and continue to develop them throughout the children's school careers. Schools should discuss how they could develop these necessary qualities, which include the following.

- The ability to discuss, work cooperatively and work individually.
- The ability to communicate using mathematics.
- The ability to define and understand problems.
- The ability to think of key questions.
- The ability to explore and experiment.
- The ability to recognise 'blind alleys'.
- The ability to develop 'transfer skills'.
- The ability to use imagination and flexibility of mind.
- The ability to be reflective.
- The ability to persevere.

Mathematical problem solving skills

In addition to the generic qualities listed above, the National Curriculum lists these skills.

Using and applying number

Pupils should be taught to:

Problem solving

a) Approach problems involving number, and data presented in a variety of forms, in order to identify what they need to do.
b) Develop flexible approaches to problem solving and look for ways to overcome difficulties.
c) Make decisions about which operations and problem-solving strategies to use.
d) Organise and check their work.

Communicating

e) Use the correct language, symbols and vocabulary associated with number and data.
f) Communicate in spoken, pictorial and written form, at first using informal language and recording, then mathematical language and symbols.

Reasoning

g) Present results in an organised way.
h) Understand a general statement and investigate whether particular cases match it.
i) Explain their methods and reasoning when solving problems involving number and data.

The lessons

Each lesson follows the same format.

Learning objectives
These are taken directly from the Yearly teaching programmes in the *Framework for Teaching Mathematics*. The solving problems objectives are linked with at least one other objective.

Vocabulary
This lists all the appropriate words and phrases to be used in the lesson. It is vital that children should see these words as well as hear them. So they should either be written on the board or if sets of vocabulary cards are available use these.

Resources
The lessons have been written to use a minimum of resources. Most of the resources listed would be found in most primary classrooms. Some lessons have resource sheets or activity sheets that can be photocopied.

Oral and mental starter
These short sessions are intended to provide the children with a lively and fun start to the lesson. The objectives are taken from the National Numeracy Strategy's sample medium-term planning.

Teaching points
A detailed lesson plan to guide you through the lesson. The emphasis is on lively activities that will demonstrate to the children that mathematics is alive!

Plenary
All the plenaries have been planned to allow the children to reflect on what has gone before. Often there are 'challenges' included in the plenary. The principle here is to ensure that the children's ability is challenged but in a non-threatening way.

Support and Extension
These sections are aimed to support the less able and challenge the more able. The nature of problem solving is such that much of the work is 'open ended' and therefore differentiation should be more manageable.

Questions to guide assessment
Teacher assessment is an important component of teaching. These questions are included to help you focus on a small number of issues. The nature of problem solving is such that it is very difficult to make summative judgements. For example, 'Presents results in an organised way'. The child in Reception can do this in one way and a Y6 child in another way but both could be as valid. It is up to teachers to use their professional judgement through teacher assessment.

Calculators

Some of the lessons involve the use of calculators and in particular the OHP calculator. This is a very powerful learning tool for young children.

Wherever the calculator is suggested it is used to support children's learning. Therefore the approach in these books is in line with the recommendations of the National Numeracy Strategy.

RECEPTION

How many ways?

Framework for Numeracy objectives

Solving problems: Reasoning about numbers or shapes

- Solve simple problems or puzzles in a practical context, and respond to 'What could we try next?'

Counting and recognising numbers

- Count reliably up to 10 everyday objects.

Measures, shape and space

- Use everyday words to describe position.

VOCABULARY

above, below, bottom, check, count, cube, different, how many ? side, square, ten, top, put together

Resources

- Cubes
- Photocopiable Sheets 1 to 4 (pages 42 to 45)
- Blu-tack
- Number lines

Oral and mental starter

Objective: Count reliably up to 10 objects

- The oral and mental starter is included in the main part of the lesson.

Problem-solving challenge

How many different ways can 10 cubes be arranged on squared paper?

With the whole class

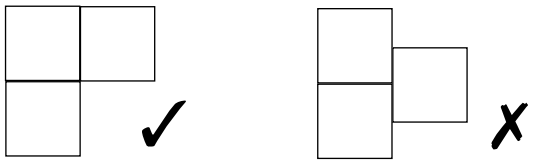
- Explain to the children that today's challenge is to try and find as many ways as they can of putting 10 cubes on to squared paper to make shapes. Demonstrate this by asking a child to count out 10 cubes at the front of the class. Ask the rest of the children to count aloud in unison.
- Stop the children at certain points and ask questions such as:

  'What number comes next?'

  'What number is 2 more than where we are now?'
- Ask the child holding the cubes to cover them up. Ask questions like:

  'How many cubes are hidden?'

  'If I take 1 cube away how many cubes will I have then?'
- When 10 cubes are counted out, select a few children to put Blu-tack on them. Demonstrate how to fix them on the large squared paper. The rule is that the cubes have to be placed together exactly side by side.



  Ask the children to count the cubes as you point to them.
- Invite a couple of children to come out and arrange the cubes in different ways. Ask:

  'Are you going to put them all in a straight line?'

  'Are you going to make a shape with them?'

Children working in pairs

- Tell the children that they have to work in pairs to find as many different ways as they can to arrange the 10 cubes on the squared paper. Give them copies of photocopiable Sheet 1 (page 42) and ask them to draw coloured dots on the squared paper to show where the cubes went. Then ask them to draw a coloured line around the complete shape that they made with their cubes. Encourage them to invent names for their shapes!
- Tell them that, to make sure the rest of the cubes don't feel lonely, they have to use a different 10 cubes for each shape! This will give them plenty of practice in counting. Ask:

  'How can you check that you have got 10 cubes?'

- It is good practice for children to get into the habit of checking. Ask:

  'How many different ways could there be of arranging the 10 cubes?'

- Go around the class to ensure that the children understand the activity and are doing it correctly.
- When they have found enough ways, give them either Sheet 2, 3 or 4 (pages 43 to 45), according to their level of achievement. Tell them to count the number of cubes in each shape on the sheet and write the answer next to the shape.
- The activity sheets will act as a guide to assessing the children's ability to count to 10 and beyond.

Plenary

- Choose some children to show their sheets. Ask questions such as:

  'How many shapes did you get?'

  'What did you call them?'

  'Has anyone got any more?'

  'How did you check that you had got 10 cubes each time?'

  'Did anyone else check in a different way?'

- Talk about the shapes using correct language such as 'long', 'wide' and 'symmetrical'. Explain the meaning of these words as you go along.
- Establish that there are lots of ways to arrange the 10 cubes. Ask:

  'Did you always count in ones?'

  'Are there other ways you can count them?'

  The idea is that the children begin to use different strategies to count, such as in twos. Some might recognise that a square shape of four cubes will always be four cubes, so they can put four in their heads and count on.

Support

- Limit the number of cubes to a number that the children can comfortably count up to. They should then be able to join in with the same activity as the rest of the class.
- Write the following letters on the board: L E I H T. Ask the children how many cubes they would use to make shape L on squared paper. They can repeat this with the other letters.
- These children should do photocopiable Sheet 2.

Extension

- Use as many cubes as the ability of the children allows.
- Let the children use as many cubes as they want and set the challenge of finding as many square shapes as they can with their cubes.
- These children should do photocopiable Sheet 4.

Questions to guide assessment

- How did the children count the cubes?
- Did they count accurately?
- Did they check their counting and, if so, how?
- Did any of the children use a simple strategy to try to solve the challenge?

How many wellies?

Framework for Numeracy objectives

Solving problems: Problems involving 'real life'

- Use developing mathematical ideas and methods to solve practical problems involving counting and comparing in a real context.

Counting and recognising numbers

- Count reliably up to 10 everyday objects.

Adding and subtracting

- Find one more or one less than a number up to 10.

VOCABULARY

count, less than, more than, pair

Resources

- Newspaper of differing sizes
- Shoes or 'indoor shoes' or slippers or wellington boots
- Photocopiable Sheets 5 to 7 (pages 46 to 48)
- Number lines

Oral and mental starter

Objectives: Count reliably to 10; count in twos; count backwards from a given number

- Play 'Take off'. The children sit in a circle and have to count around the group. When they get to 10, instead of saying '10' the child says/shouts, 'Take off', and stands up. The next child starts at 1 again and so on.
- Play this for a while and then see if the children can play the game counting up in 2s.
- As an extension to this, ask the children to count backwards from 10, saying 'Take off' instead of '1'.

Problem-solving challenge

How many wellies can fit on a sheet of newspaper?

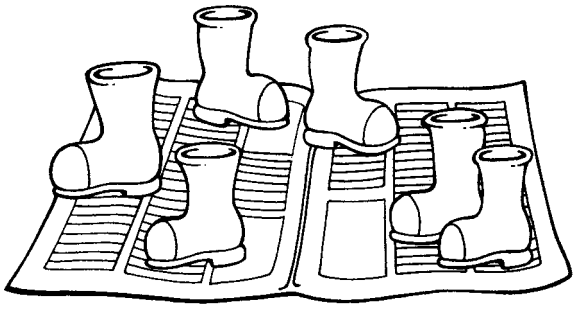
With the whole class

- For the purposes of this lesson, the problem is to do with wellington boots but it could easily be about shoes, slippers and so on.
- Set the scene with the children by explaining how people like to put wet wellies on newspaper to dry and if you have a lot of wellies you would need a lot of newspaper!
- With the newspaper and wellies, ask some children to come out to the front of the class and try to find different ways of putting the wellies on the newspaper. The rule is that no wellies are allowed to overlap the edges of the newspaper. Always ask the children to count how many fit on. Every time they arrange the wellies, ask lots of questions such as:

  'How many wellies are there?'

  'How many wellies would there be if I took 1 off?'

  'How many wellies would there be if I took 2 off but then put 1 back?'



Children working in small groups

- The children should continue to investigate how many different ways they can find of fitting the wellies on the newspaper. In the first instance, give them tabloid-sized newspapers to work with. Later, you can give them broadsheet papers. Encourage them to think of 'more than' and 'less than' questions and statements about the activity. For example:

  'If I fit the boots round this way, there are more on the paper than if I fit them this way.'

  'Are there less wellies on that paper than on this one?'

- When the children have explored the possibilities, give them the relevant photocopiable sheet from pages 46 to 48. Tell them they are to count the number of wellies on each sheet of newspaper and write the answers in the boxes. These activities will act as a guide to assessing the children's ability to count to 10 reliably.

- Where necessary, you may need to help children with their recording. An alternative is to organise the children into maths pairs and let them discuss the problem and one of them record the answer.

Note
Have a number line on display for the children to refer to. Let them come and count along it if they need to.

Plenary

- Ask each group of children what strategies they used to try and fit the wellies on the sheet of newspaper. Ask them to say some of their 'more than' and 'less than' questions and statements.

- Explain to the children what a pair is. Give them some questions relating to pairs and let them discuss in pairs (!) the answer to, for example:

  'How many pairs of wellies would there be if we had four wellies?'

Support

- The best way to support the less able in this lesson is to group them with other, more able children who can help them with the counting if necessary. It is important to ensure that any more able children are not dominating the group.

- These children should work in pairs or small groups and complete photocopiable Sheet 5.

Extension

- Ask the children to investigate how many pairs of wellies could fit on the newspaper.

- These children should be able to work independently and complete photocopiable Sheet 7.

Questions to guide assessment

- Could the children count their wellies accurately?
- Did they work cooperatively?
- Could they explain their methods and how they solved the problem?
- Could they think of key questions?

RECEPTION

Sort the class

Framework for Numeracy objectives

Solving problems: Reasoning about numbers or shapes

- Sort and match objects, pictures or the children themselves, justifying the decisions made.

Counting and recognising numbers

- Count reliably up to 10 everyday objects.

VOCABULARY

count, equals, less, match, more, sort

Resources

- Children!
- Objects to match
- 10 pencils, 8 rulers and so on
- Photocopiable sheets 8 to 10 (pages 49 to 51)

Oral and mental starter

Objective: Begin to relate addition to combining two groups of objects and then counting all the objects, then extend to three groups of objects

- Ask four children to come out and stand in two groups of two. Ask the class to count how many children are in the two groups.
- Explain that another way to find the answer would have been to say, '2 add 2 equals 4'.
- Repeat this with other groups of two and then extend it to groups of three.

Problem-solving challenge

With the whole class

- This lesson may need to be done in the hall or other large space.
- Without telling the children, choose a criterion such as brown hair and, name by name, ask all the children with brown hair to stand up and put themselves into a group. Ask the rest of the class:

  'Why have I chosen all these children?'

- Be prepared for some interesting answers! You will probably have to give them lots of helpful hints.
- When, the children eventually guess the criterion, ask how many children are in this group. If the number of children is large, help with the counting.
- Repeat this activity with lots of other 'secret' criteria – for example, the colour of tops, trousers or skirts.
- Explain to the children that you have been doing something called 'sorting'.
- Ask a child to come out and whisper a secret criterion to you and then choose children that match it to stand up. Beware of any criteria that may be offensive, such as 'fat children' or 'children I don't like'. Repeat this sorting activity with other criteria.
- Stand up and tell the children that you are a group of one! Ask them to think of what criteria there might be for you being a group of one – for example, you are an adult.
- Explain that you are going to work together to 'match' some objects to some children. Ask a group of children to hand out objects such as pencils. Make sure that there are fewer pencils than there are children. Ask questions such as:

  'How many more pencils do we need so that everyone can have a pencil?'

- Repeat this with different objects, varying the number of objects, sometimes having more objects than children. All the time, encourage the children to use the correct language, for example 'more than' and 'less than'. For example:

  'There are more children than pencils.'

Children working in small groups

- Give the children the relevant photocopiable sheet from pages 49 to 51. They have to match the groups of numbers by drawing lines.
- These sheets can then act as an individual assessment.

Plenary

- Without telling the children what they are, choose two criteria, for example blonde hair and glasses. Ask those children who fit these criteria to stand up. Ask the class if they can guess why these children are standing up. Explain what you have done and ask someone if they can come out and choose two secret criteria as before.
- Remind the children what the words 'sort' and 'match' mean. Ask:

  'Richard, can you match three pencils to some children so the children have one each?'

  'Maya, can you sort the crayons into a red pile and a blue pile?'

Support

- For the first part of the lesson, ask a child to come to you and whisper to them a criterion and then help them to choose the children that fit that criterion.
- In the matching part of the lesson, involve the children by targeted questioning, for example:

  'Do you think that is more than or less than?'

- These children can then complete photocopiable Sheet 8.

Extension

- Encourage the children to think of a really hard criterion for sorting children, such as boys with brown hair and brown eyes. Ask targeted questions to challenge them, such as:

  'What would the number be if it was twice as many?'

- These children can then complete photocopiable Sheet 10.

Questions to guide assessment

- Could the children count reliably?
- Could the children sort by one criterion?
- Could the children match one-to-one?

How big is big?

Framework for Numeracy objectives

Solving problems: Problems involving 'real life' or money

- Use developing mathematical ideas and methods to solve practical problems involving counting and comparing in a real or role-play situation.

Measures, shape and space

- Use language such as 'more or less', 'longer or shorter', 'heavier or lighter' to compare two quantities.

VOCABULARY

bigger, biggest, longer, longest, smaller, smallest, thinner, thinnest, thicker, thickest, taller, tallest, wider, widest

Resources

- Vocabulary cards made from photocopiable Sheet 11 on page 52
- Objects to compare
- Photocopiable Sheet 12 (page 53)

Oral and mental starter

Objective: Count in twos

- Sit the children in a circle and ask them to count around the group in ones. Let them count as high as they can cope with. Repeat this, but this time every time a child counts a multiple of two that child has to stand up. With the children still standing up, get the class to just count those children standing up in twos.
- Repeat this with the 'twos' children standing up as fast as they can.

Problem-solving challenge

What is the biggest thing in the school?

With the whole class

- Tell the children that they are going to decide what is the biggest thing in the school. What do they think 'biggest' means?
- First talk about longest and tallest. Show them two objects, such as a teaspoon and a fork, and ask:

  'Which one is longer?'

  When they are agreed, find the vocabulary card (from sheet 11, page 52) that says 'longer' and show it to them.
- Add a wooden spoon to the teaspoon and fork. Ask:

  'Which one is the longest?'

  Explain that when there are two we say 'longer', but when there are more than two we say 'longest'. Find the vocabulary card.
- Ask two children to stand up. Ask:

  'Who is the taller?'
- Ask another child to stand up. Ask:

  'Who is the tallest?'

  Again, explain that when there are two we say 'taller', but when there are more than two we say 'tallest'. Find the vocabulary card.
- Introduce the words 'wider' and 'widest' using objects such as tables or spaces between objects to demonstrate what these mean.
- Now introduce the idea of 'big'. What do the children think of as 'big'? (Buildings, aeroplanes, dinosaurs, elephants and so on?) Talk about 'bigger' and 'biggest'. Explain that it is not necessarily the tallest, or the longest, or the widest.

Children working in small groups

- Tell the children that they are going to work in pairs to discuss what they think is the biggest thing in the classroom and then the biggest thing in the school. They can record what they think in words, drawings, or both, on Sheet 12 (page 53). Tell them to keep their decisions a secret from the other pairs.

Plenary

- Ask all the children to fold their sheets in half and hold up the half that shows what they think is the biggest thing in the classroom. Ask individuals why they think this. Encourage them to use the correct vocabulary and hold up the vocabulary cards as they do so. For example:

  'Did you think the blackboard is biggest because it is widest or tallest?'

- Then ask the children to hold up the halves of their sheets that show what they think is the biggest thing in the school. How many agree? Take them on a walk around the school to try to find the biggest thing. When you have found it and agreed that it is, ask:

  'Was anyone right?'

- When you are all back in the classroom, discuss the different words the children have been using. Talk about how the words 'bigger' and 'biggest' are not very specific and that terms like 'taller' or 'wider' are often more appropriate.

- Introduce the vocabulary cards that they haven't so far used. Talk about the words and what they mean.

- Give the children a challenge to think about for tomorrow:

  'What do you think is the biggest thing in the world?'

Support

- Give the children cards made from photocopiable Sheet 11 (page 52). They put the cards face down and play a game where they turn two cards over and match them, for example 'taller' and 'tallest'. If you have a classroom assistant he or she could use the cards as 'flash' cards for the children to read out.

Extension

- Challenge the children to say exactly what certain words mean, for example:

  'What does "tall" mean?'

Questions to guide assessment

- Could the children use the correct vocabulary?
- Could the children compare accurately?
- Did the children work cooperatively to tackle the problems?